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Quantum teleportation and the non-locality of information

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A two-state quantum system (a qubit) can carry one bit of classical information. If two bits are not encoded into two qubits separately, but only into their joint properties, entangled states result. It is shown explicitly how quantum teleportation uses this feature and how the randomness of the individual quantum event prohibits instantaneous communication.

1. Introduction

Consider encoding information into the polarization of photons. Suppose we agree to identify the horizontal polarization of the photon with '0' and the vertical polarization of the photon with '1'. This means we choose linear polarization as our computational basis. Thus $|H\rangle \equiv |0\rangle$ and $|V\rangle \equiv |0\rangle$. Encoding the maximum possible two bits of information into two photons can then be done easily using the four factorizable states $|H\rangle|H\rangle$, $|H\rangle|V\rangle$, $|V\rangle|H\rangle$ and $|V\rangle|V\rangle$. It is clear that in this complete basis of the four-dimensional Hilbert space of the two photons, each photon's polarization, and thus the information each photon carries on its own, is well defined. The information content can thus be expressed as the truth values of the following two statements: 'The polarization of the first photon is vertical'; 'The polarization of the second photon is vertical'.

Thus, the information is expressed in clear statements about each photon's private properties.

Yet, from quantum mechanics we know that there are other possible ways the photon pair can carry information. Consider, for example, the statement: 'Both photons have the same polarization'.

Then the state of the two photons could be either $|H\rangle|H\rangle$ or $|V\rangle|V\rangle$. Suppose the experiment is such that it cannot distinguish between $|H\rangle|H\rangle$ and $|V\rangle|V\rangle$, then the quantum state has to be the general superposition $1/\sqrt{2}(|H\rangle|H\rangle + e^{i\varphi}|V\rangle|V\rangle)$, where φ is not fixed by the information given thus far. Considering for simplicity just the possibilities $\varphi = 0$ and $\varphi = \pi$, we arrive at the two orthogonal basis states

$$\left.\begin{aligned} |\Phi^+\rangle &= 1/\sqrt{2}(|H\rangle|H\rangle + |V\rangle|V\rangle) = 1/\sqrt{2}(|0\rangle|0\rangle + |1\rangle|1\rangle), \\ |\Phi^-\rangle &= 1/\sqrt{2}(|H\rangle|H\rangle - |V\rangle|V\rangle) = 1/\sqrt{2}(|0\rangle|0\rangle - |1\rangle|1\rangle). \end{aligned}\right\} \tag{1.1}$$

Alternatively, the information we have might be: 'Both photons have different polarization'.

Then by similar reasoning we arrive at the two basis states

$$\left.\begin{aligned} |\Psi^+\rangle &= 1/\sqrt{2}(|H\rangle|V\rangle + |V\rangle|H\rangle) = 1/\sqrt{2}(|0\rangle|0\rangle + |1\rangle|0\rangle), \\ |\Psi^-\rangle &= 1/\sqrt{2}(|V\rangle|H\rangle + |V\rangle|H\rangle) = 1/\sqrt{2}(|0\rangle|0\rangle - |1\rangle|0\rangle). \end{aligned}\right\} \quad (1.2)$$

The states (1.1) and (1.2) together form the complete Bell basis of maximally entangled states.

It is important to note that thus far we have only used one bit of information for the pair. This bit is the truth value of the statement 'Both photons have different polarization'. Since we have used only one bit, we still have the ambiguity between the two states in equation (1.1) and the two states in equation (1.2), respectively. How can we stop this ambiguity? Or, in other words, what is the second bit of information we need to fully characterize the Bell basis? What we search for here is not a formal definition as, for example, the different sign between the terms in the Bell-states, but an operational distinction which can immediately be applied in experiment.

A second bit of information about the Bell basis states can be found if we represent these states in terms of the basis states of circular polarization $|R\rangle = 1/\sqrt{2}(|H\rangle + \mathrm{i}|V\rangle)$ and $|L\rangle = 1/\sqrt{2}(|H\rangle - \mathrm{i}|V\rangle)$. Omitting here, and also in the paper, unimportant overall phase factors of states, we find

$$\left.\begin{aligned} |\Phi^+\rangle &= 1/\sqrt{2}(|R\rangle|L\rangle + |L\rangle|R\rangle), \quad |\Phi^-\rangle = 1/\sqrt{2}(|R\rangle|R\rangle + |L\rangle|L\rangle), \\ |\Psi^+\rangle &= 1/\sqrt{2}(|R\rangle|R\rangle - |L\rangle|L\rangle), \quad |\Psi^-\rangle = 1/\sqrt{2}(|R\rangle|L\rangle - |L\rangle|R\rangle). \end{aligned}\right\} \quad (1.3)$$

Thus we can readily identify the second bit of information carried by the maximally entangled Bell basis as expressing the truth value of the statement: 'The two photons have different circular polarization'. And the statement about the first bit has to be more precisely formulated as 'The two photons have different linear polarization'.

It is evident that these two bits of information have immediate operational meaning, and one can use them to characterize uniquely the four Bell-states.

This brief discussion sheds light on the operational meaning of entanglement. In our case of two entangled particles, each one in a two-dimensional Hilbert space, the information which can be carried jointly by both particles is two bits. In the maximally entangled Bell-states, this information is only expressed in terms of relational properties of the two photons between each other. There are two different such relational statements exhausting the two bits which can be carried by two photons. Therefore, no information is carried by each photon on its own. The complete encoding scheme is shown in table 1.

2. Quantum teleportation

In the quantum teleportation scheme proposed by Bennett *et al.* (1993), the physics of entangled states is directly applied. The problem is that Alice wants to teleport an unknown quantum state $|\Psi\rangle$ to Bob such that Bob has identically the same state at hand. For some reason we assume that it is impossible that Alice sends the state directly to Bob. The proposed solution (figure 1) is to use an entangled ancillary pair and perform a joint Bell-state measurement on $|\Psi\rangle$ and one of the members of the ancillary. This Bell-state measurement projects these two photons onto one of the maximally entangled states of the Bell basis, resulting in four possible outcomes Φ^+, Φ^-, Ψ^+ or Ψ^-. That is, the information we obtain is just the two bits encoded in

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Table 1. *Encoding of two bits of information into the Bell-states*
(All information is contained in the entanglement.)

			linear polarization	
		bit	equal '0'	different '1'
circular polarization	equal	'0'	$\|\Phi^-\rangle$	$\|\Psi^+\rangle$
	different	'1'	$\|\Phi^+\rangle$	$\|\Psi^-\rangle$

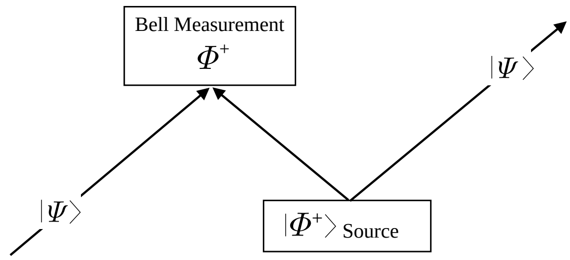

Figure 1. Instant quantum teleportation. Alice teleports an unknown quantum state $|\Psi\rangle$ by performing a joint Bell-state measurement on $|\Psi\rangle$ and on one of the two photons of an ancillary entangled pair. If Alice obtains the result Φ^+, we have the apparent relativistic paradox that Bob obtains Alice's state $|\Psi\rangle$ instantly or even retroactively in time.

the Bell basis. The important point now is that this measurement also projects the other member of the ancillary pair which is heading towards, Bob into one of four unique states.

Let us consider for simplicity that the ancillary pair is in the state $|\Phi^+\rangle$ and let us assume that the result of Alice's Bell-state measurement happens to be Φ^+. In that case Bob has to apply the identity operator, i.e. he does not need to do anything in order to obtain $|\Psi\rangle$. Then we might very well argue that Bob had the unknown state $|\Psi\rangle$ already in his possession at a time before Alice obtained her result Φ^+. Thus we have apparently a paradoxical situation for two reasons. Firstly, Bob seems to obtain the information contained in the unknown state $|\Psi\rangle$ at times violating all relativity considerations. And secondly, for the time before Alice's Bell-state measurement, we seem to have two identical copies of $|\Psi\rangle$, violating the no-cloning theorem (Wooters & Zurek 1982). Yet this paradox is only apparent because Bob needs the classical information that Alice really obtained the result Φ^+, in order to know that he has an exact copy of $|\Psi\rangle$ in his hand. It could very well be that Alice obtains another of the four results in which case Bob needs to apply a specific unitary transformation, different in each case, to obtain $|\Psi\rangle$.

Colloquially speaking, it is fair to say that quantum mechanics provides a possibility to send signals faster than the speed of light or even retroactively in time at the expense of the fact that the recipient of the information cannot read the information. He does not know whether what he gets is faithful information or not. Bob might actually try to take advantage of the fact that in one of four cases he really has the state $|\Psi\rangle$ instantly at hand by simply guessing when this might be the case. Should his guess be correct, he actually had information faster than the speed of light. Clearly, the problem is that whatever guessing strategy Bob adopts, he can never be better than any random method. We thus see how the randomness of the individual quantum event, as expressed by the randomness of the result obtained by Alice, prohibits teleportation to violate special relativity. This example also underlines clearly that the speed of light is the limiting velocity of information and that this information can be of a rather subjective nature, as in our example, because objectively we know that Bob can have the state $|\Psi\rangle$ instantly in his hands.

Our little example also shows how quantum teleportation nicely underlines the nature of information assignment in entangled systems. Information assignment is actually used twice in the scheme, both for the preparation of the ancillary pair and in Alice's Bell-state measurement. In our simple example, the two statements are about the relative properties of the two photons in the ancillary pair and about the relative properties of the unknown photon and one of the ancillaries. There is no statement whatsoever made about the properties of the teleported quantum state $|\Psi\rangle$. This is actually necessary because no measurement of $|\Psi\rangle$ could provide enough information to characterize sufficiently the quantum state itself. Therefore, in a very elegant way, quantum teleportation eschews the limit given by the uncertainty principle, which states that no measurement of a quantum system can reveal enough information to fully characterize it.

We conclude this brief note by making two remarks. Firstly, in order to characterize our two bits of information, we could take any pair of basis states related to each other by a 90° rotation on the Poincaré sphere. Secondly, the teleportation experiment is presently under way at our laboratory in Innsbruck. The main experimental problem is the joint projection of the photon to be teleported, and one of the ancillary photons onto the Bell basis. While no complete Bell-state measurement procedure exists, we can experimentally identify three of the four Bell-states (Mattle *et al.* 1996) using just beam splitters and polarizers. Furthermore, projection onto the Bell basis entails complete lack of knowledge about where a specific detected photon originated. This implies very detailed quantum erasure protocols (Zukowski *et al.* 1993, 1995; Rarity *et al.*, this volume).

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